Introduction to Functions

A relation is any set of ordered pairs.

For example, we could pair each person’s first name with their age: (Suzie, 21), (Kyle, 19), (Angie, 28) and so on. This list is a relation; it is a set of ordered pairs.

Relations can be described in 5 ways:

Example 1:

1) a verbal description
   “The set of all points 5 units from the origin.”

2) a table
   A table of some points:

<table>
<thead>
<tr>
<th>x</th>
<th>0</th>
<th>0</th>
<th>3</th>
<th>3</th>
<th>-3</th>
<th>-3</th>
<th>5</th>
<th>-5</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>5</td>
<td>-5</td>
<td>4</td>
<td>-4</td>
<td>4</td>
<td>-4</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

3) a graph
   Graph the relation:

   ![Graph of a relation]

4) set notation
   A possible set is { (0, 5), (0, -5), (3, 4), (3, -4), (-3, 4), (-3, -4), (5, 0), (-5, 0) }  

5) an equation
   The equation is \( x^2 + y^2 = 25 \).  

A function is a relation in which, for each value of the first component of the ordered pairs, there is **exactly one** value of the second component.

A function is a relation in which, in the ordered pairs, for each x-value, there is **exactly one** y-value.

A function is a relation in which every input has exactly one output.

The age example is a function because each person can have only one age (second component or y-value).

**In a function, no two ordered pairs can have the same first component and different second components.**

(one person can’t be two different ages)

A function is a rule that produces a correspondence between the first set of elements called the **domain** (independent variable or input or x’s) and a second set of elements called the **range** (dependent variable or output or y’s) so that to each element in the domain there corresponds **one and only one** element in the range.

**Back to Example 1:** Is the set of all points 5 units from the origin a function? ______

**Example 2:** Is $y = 3x - 4$ a function? ______

Name the domain in interval notation (set of all possible inputs) ______________

Name the range in interval notation (set of all possible outputs) ______________
**Agreement on Domain:** Unless specified otherwise, the domain of a relation is assumed to be all real numbers that produce real numbers when substituted for the independent variable (usually $x$).

This means in an equation like $2x - 5y = 10$ any real number can be used as a replacement for $x$.

But in $y = \frac{1}{x}$, $x \neq 0$ because the fraction would be undefined. The domain is all real numbers except 0.

What’s special about square roots?
An even root of a negative number is not a real number (for example, $y = \sqrt{-9}$ is not a real number).
**Example 3:** How about \( y = \sqrt{x - 2} \)? What’s the domain?

\( x - 2 \) must equal a positive number or 0.

Our domain is ___________________________________________

There have been 2 key ideas so far:

1. A **function** is a relation that assigns to each element in the domain \((x)\) **exactly one** element in the range \((y)\).

2. The **domain** is the set of all possible inputs and the **range** is the set of all possible outputs.

One way to identify a function is by the **Vertical Line Test**. If a vertical line passes through the graph of a relation in no more than one point, then the relation is a function.

The example of the circle you graphed on the first page is not a function. It does not pass the vertical line test.
Example 4: Does this relation represent a function?

Example 5: Determine if $y = \sqrt{3x - 2}$ is a function and state the domain in interval notation.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Is it a function? ________

Domain _________________
Explore functions, domain and range before going on:

Function: \( y = 3 \)  
Function: \( y = x \)  
Function: \( y = |x| \)

Domain:___________  
Domain:___________  
Domain:___________  

Range:___________  
Range:___________  
Range:___________
Function: Not a function: an ellipse Not a function:

Domain:___________ Domain:___________ Domain:___________

Range:___________ Range:___________ Range:___________
Using Function Notation

When a function is defined with a rule like an equation with $x$ dependent on $y$, we use function notation to emphasize that dependence. The notation is $y = f(x)$ and is read “$y$ is equal to $f$ of $x$.” The letter $f$ stands for function. In other words, in a function, we solve for $y$ and replace $y$ with $f(x)$. This states that it is a special equation we call a function.

Example 6: Given the function $3x + y = 5$, restate it using function notation.

First solve the equation for $y$ then replace $y$ with $f(x)$.

If we want to find the value of the equation in example 6 when $x = -1$ we say find $f(-1)$. This means substitute $x = -1$ into the equation and find the value of the function.
Example 7: Let \( g(x) = \frac{-3x + 5}{2} \). Find \( g(-3) \)

Example 8: Let \( h(x) = 5x - 1 \). Find \( h(1), h(-6), h(n) \) and \( h(l + 2) \).
Example 9: Rewrite $y - 3x^2 = 2$ in function notation and find $f(-1)$ and $f(a)$.